Dissertation Weekly Progress Report
Week 5

Date of Last Meeting: 29 October 2026
Date of Next Meeting: 5 November 2026 ✳
Current Focus Area: Bringing It All Together, More CAMB Experimentation, Next Steps

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Another busy week! I fear my reports are dropping in quality as the weeks progress, so I will try not to make a habit of this.

1 'Introduction' to the Project

Maybe it is worth starting to write an overview (introduction-style) to what I have done so far which can be used in the interim report at the end of this term. Most of the content I will present in this section I already knew upon starting the project, but I think it useful to dedicate a little time to writing this out now, such that when I get into deeper research near the end of my project I can focus more on that, and have some of the 'padding' already written. The inclusion of modern papers in adjacent fields that often populate such introductions will be added at a later date, as well as citations to back up my historical/observational claims.

This report will include a introduction and a review of the need for inflation (through the Horizon and flatness problem).

The scientific study of the Universe, that is Cosmology, is of great importance in furthering our understanding of fundamental physics. Indeed, the very early Universe [1] is the best laboratory for high energy physics, probing energies otherwise inaccessible to our current capabilities. By identifying and exploring late-time signatures (signatures observable *today*) we can inform our theories by quantifying the effect we expect to see. Such investigations require a full and accurate knowledge of the evolution of the universe between its 'beginning' and the current time and thus we must start with a comprehensive theory for the very early universe.

Let us review the most intrinsic facts of our Universe that we have gathered, spurred on by the current era of 'precision cosmology'. Firstly, we have that the universe on the largest scales appears homogeneous and isotropic, the so-called *cosmological principle*. The most ubiquitous example of this is in the Cosmic Microwave Background (CMB) radiation, where the relative temperature variation across the entire sky (after removal of systematics and the dipole anisotropy) is roughly 1 part in 100,000. Secondly, the universe is undergoing expansion (specifically, metric expansion) with recession velocity between un-bound systems being proportional to their distance (this is *Hubble's Law*). This motivates the idea that the Universe has a 'beginning'; if it is expanding, it must have been smaller in the past. A more surprising obervation is the fact that the universe is *accelerating* in it's expansion. This is not what we would expect from a matter-dominated universe, and thus motivates us to devise a mechanism by which the universe is *dark-energy-dominated*, the leading contender for which is a cosmological constant. Finally, we have observed that the Universe is very close to *spatially-flat*, which has implications we will discuss futher in [ADD REF]. These observations are formulated into a theory known as ΛCDM, where the Universe is endowed with a Friedmann-Lemaître-Robertson-Walker (FLRW) metric where the expansion is quantified by a *scale factor* $a(t)$[2].

The ΛCDM model does not, however, offer any explanation of the initial conditions of the Universe. It offers only an approach to quantify the evolution of the Universe from some initial

[1] The capitalisation of 'Universe' refers to it being our universe, rather than some other (theoretical) model universe.

[2] Here t, refers to *cosmic time* which is measured in a comoving-frame at rest with respect to the CMB.

state, that is assumed to be seeded with primordial density fluctuations of a certain form [3]. The leading explanation of where these primordial perturbations came from is the theory of *Inflation*, which posits a cosmological epoch of exponential expansion 'before' the era of reheating. Before we discuss Inflation futher, let us first look at why such an epoch is necessary to better explain the observations we listed above.

1.1 The Horizon Problem

Before we discuss the Horizon problem, it is necessary to introduce some relevant terminology.

First we define the Hubble Parameter as $H(t) = \dot{a}(t)/a(t)$. From this, we define the Hubble time as $H^{-1}(t)$ and the Hubble length as cH^{-1}. Throughout our work we will choose the units $c = 1$ and so the Hubble length becomes H^{-1} and we call it the **horizon**, and it acts as an estimate of the distance light can travel during a Hubble time.

Before going further, let us review comoving coordinates. In this formalism, comoving distance is constant during universal expansion. Thus, a comoving coordinate r can be written in terms of a physical coordinate (one that is observable) as $r = 1/a(t)r_{\text{phys}}$ or, in a more common form, $r_{\text{phys}} = a(t)r$. In addition to this, it is often useful to introduce *conformal time* as $\mathrm{d}\eta = \mathrm{d}t/a(t)$.

Now we have all we need to describe if a region of size $Ra(t)$[4] is inside or outside the horizon. Such categorisations become important not only in the discussion of this problem, but in the later study of *horizon freezeouts*. The sub-horizon (within the horizon) or super-horizon (outside the horizon) categorisation of regions is given by the ratio

$$\frac{Ra}{H^{-1}} = RaH = R\dot{a}.$$

If this ratio is less than one, the region is sub-horizon and, if greater than one, the region is super-horizon.

We can also define the **particle horizon** by

$$d_h(\eta) = \eta - \eta_i = \int_{t_i}^{t} \frac{\mathrm{d}t}{a(t)}$$

. If one sets the initial time (we call this point the 'Big Bang', but we should not worry too much about the physical connotations of this naming) as $\eta_i \coloneqq 0$ we get simply $d_h(\eta) = \eta$. The particle horizon is a measure of the comoving distance that light could have travelled since the beginning of the Universe (taken to be when $a = 0$).

If we note that the horizon H^{-1} is a physical scale, we can convert it to the **comoving horizon** by the standard transformation $H_{\text{comoving}} = a^{-1}(t)H^{-1} = (aH)^{-1}$. It is worth stating, now, what this means physically. The comoving horizon (or comoving Hubble radius) is the radius around a point at which, due to the expansion of the Universe, objects are receding at or greater than the speed of light; in essence, this gives a local boundary for interacting processes. This motivates us to represent the particle horizon in the form

$$d_h(\eta) = \int_{t_i}^{t} \frac{\mathrm{d}t}{a(t)} = \int_{a_i}^{a} \frac{\mathrm{d}a}{a\dot{a}} = \int_{\ln a_i}^{\ln a} (aH)^{-1} \mathrm{d}\ln a \tag{1.1}$$

Let us, omitting extensive explanation, present some useful results here:

- For a *radiation-dominated universe*, $a(t) \propto t^{1/2}$ and thus $H(t) = \frac{1}{2t} \propto a^{-2}$ which gives $(aH)^{-1} = (Ct^{1/2}\frac{1}{2t})^{-1} = (\frac{C}{2t^{1/2}})^{-1} = \frac{2a}{C^2}$

[3] By 'certain form' we really mean, a certain form of the *power spectrum* of the fluctuations, which we will define fully at a later point.

[4] Here R is a comoving size, which we can take to be the radius of a spherical region.

- For a *matter-dominated universe*, $a(t) \propto t^{2/3}$ and thus $H(t) \propto a^{-3/2}$

Then in the radiation-dominated limit 1.1 gives

$$\int_0^a \frac{\mathrm{d}a}{a\dot{a}} = \int_0^a \frac{1}{(Ct^{1/2})(\frac{1}{2}Ct^{-1/2})}\mathrm{d}a = \int_0^a \frac{2}{C^2}\mathrm{d}a = \frac{2a}{C^2} = (aH)^{-1}$$

We can do this similarly for the matter-dominated limit and we get that $d_h = 2(aH)^{-1}$.

Since ΛCDM is based on a radiation epoch after $n_i = 0$ which eventually transitions into a matter-dominated epoch, we can assume that in general $d_h \sim (aH)^{-1}$. Thus, the causal particle horizon only ever grows roughly as quickly as the comoving horizon. This means that regions/scales entering the horizon today, could not have been in causal contact at recombination. In other words, if a scale was outside of the horizon at recomination, it was inherently larger than the particle horizon at recombination. By the similarity of d_h and $(aH)^{-1}$, we can say that scales entering the horizon now would be making causal contact for the first time today, in the standard cosmological picture. One can also reach an understanding of the horizon problem by reasoning that since $(aH)^{-1}$ (in ΛCDM) increases monotonically, the particle horizon must have been much smaller in the past.

We solidify this idea with a more geometrical view. Suppose, as observations suggest, that $\eta_{\mathrm{rec}} \ll \eta_0$[5]. We can then draw a suggestive diagram like so:

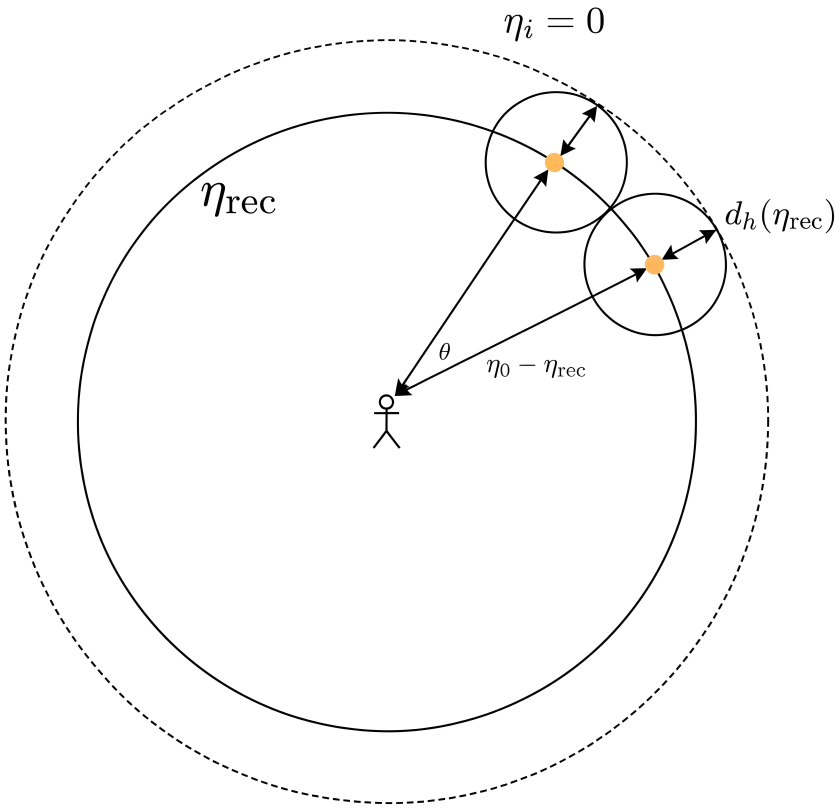


Figure 1: Geometric viewpoint of the Horizon Problem. I notice that this does not show up very well in the PDF and thus I will endeavour to improve this at a later date.

Using this, and the approximation on the relation between the conformal time at recombination and the current value, we can say that

$$\theta = \frac{2d_h(\eta_{\mathrm{rec}})}{\eta_0 - \eta_{\mathrm{rec}}} = \frac{2\eta_{\mathrm{rec}}}{\eta_0 - \eta_{\mathrm{rec}}} \approx 2^\circ$$

[5]If this is not clear, η_{rec} is the conformal time at recombination, the point at which the CMB photons scatter, and η_0 is the current conformal time.

where we have evaluated the expression using the modern values for η_{rec} and η_0. This conclusion gives that the CMB should only have a causal angular window around any point of 2°, but we see (as stated above) an extraordinary homogeneity. How could the temperature be so even across the CMB if, for example, two sides of the sky are only just entering causal connection? This seems completely inplausible and motivates an epoch where the evolution of $(aH)^{-1}$ is different to the behaviour of radiation- and matter-dominated epochs. This, as it turns out, is a characteristic behaviour of the inflationary paradigm.

1.2 The Flatness Problem

Before we properly introduce the problem that is the main topic of this section, it is worth introducing some of the most important equations in cosmology, the *Friedmann equations*:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda c^2}{3} \tag{1.2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \tag{1.3}$$

where $K \in \{-1, 0, 1\}$ is the value that determines the global geometry of the space. We also define the **density parameter** Ω as

$$\Omega - 1 \coloneqq \frac{K}{a^2 H^2} \equiv \frac{K}{\dot{a}^2}$$

. This density parameter defines spatial curvature

2 Customising the CAMB Primordial Power Spectrum

As usual on weeks where I find myself having less time to do dissertation work, I will present some coding techniques which I see might be useful in the future of the project.

2.1 Motivation

The primordial power spectrum acts as the seed of all late-time structure in the universe. ΛCDM predicts a standard nearly scale-invariant power law power spectrum, which is the standard in CAMB. Slow-roll single field inflation recovers this power-law spectrum from the Bunch-Davies vacuum. However it is not unique. We can produce theories that will give different 'shapes' to the primordial spectrum. It is therefore important, in the case of such an alteration, that we can pass this into CAMB2 to recover late-time models. This can then be used with Bayesian inference to quantify the effect of such alterations on cosmological quantities. I will present a specific example of this potential usefullness (and why, in fact, it is of no help in this regard) later. Although the closing discussion will show that recent research has proved this unsuitable to alleviate the S_8 cosmological tension, I feel this is still an important computational step, since it is a core part of working with modifications to the standard inflationary paradigm. Primordial Power Spectra, otherwise, hold an important role in understanding the physics of inflation; constraining the spectrum informs our theories of inflation itself.

An example we will use, is when there is a sharp change in the gradient of the inflaton potential we get the Starobinsky model for the resulting power spectrum [1]. This can be modelled as

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\text{st}}(k) T(k/k_*, R_*) \tag{2.1}$$

where

$$T(y, R_*) \equiv \left[1 - 3(R_* - 1)\frac{1}{y}\left\{\left(1 - \frac{1}{y^2}\right)\sin 2y + \frac{2}{y}\cos 2y\right\}\right.$$
$$\left. + \frac{9}{2}(R_* - 1)^2 \frac{1}{y^2}\left(1 + \frac{1}{y^2}\right)\left\{1 + \frac{1}{y^2} + \left(1 - \frac{1}{y^2}\right)\cos 2y - \frac{2}{y}\sin 2y\right\}\right]. \tag{2.2}$$

and looks like, for some arbitrary set of parameter values:

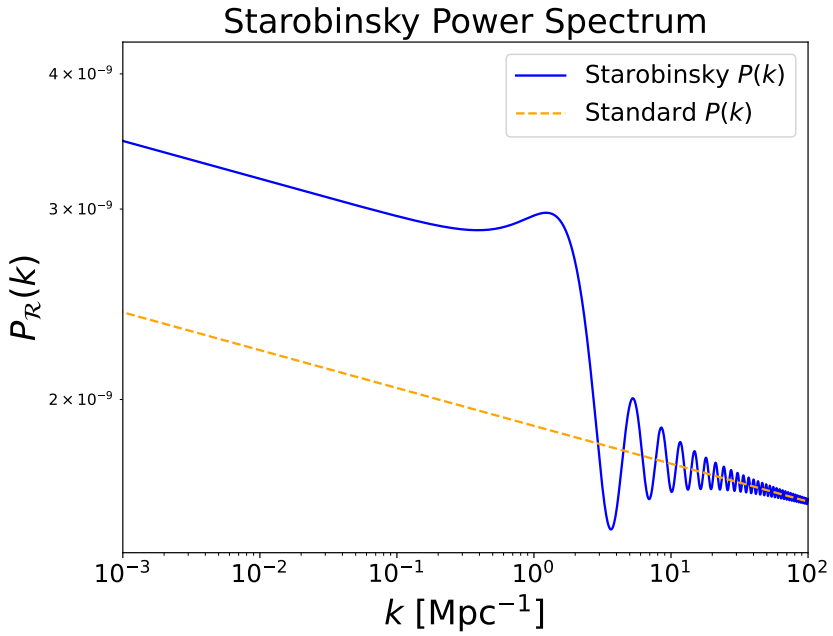


Figure 2:

In the case that $R_* = 1$ we recover the standard power-law. In modelling this spectrum we use Taylor expansions (calculated with `sympy`) to deal with numerical instability at very low scales. There is still, as far as I can tell some disagreement between this and the example in [2], but since this is purely for exploratory purposes here, I did not feel it necessary to do a full analysis of what this might be. I did however spend a fair amount of time checking the validity of the mathematical expressions and the taylor expansions. I chose not to document this work for brevity.

2.2 Late-Time Outcomes

Seeding the Universe with a Starobinsky power spectrum with parameters as seen in Table 1 we use CAMB to calculate the matter power spectrum P_{mm} and the CMB TT angular power spectrum, C_ℓ^{TT}.

Parameter	**Value**
A_s	2.1×10^{-9}
n_s	0.965
k_{pivot}	0.05
R	1.2
k_*	0.05

Table 1: Starobinsky primordial power spectrum parameters. These were chosen arbitrarily for illustratory reasons.

Here we present the results:

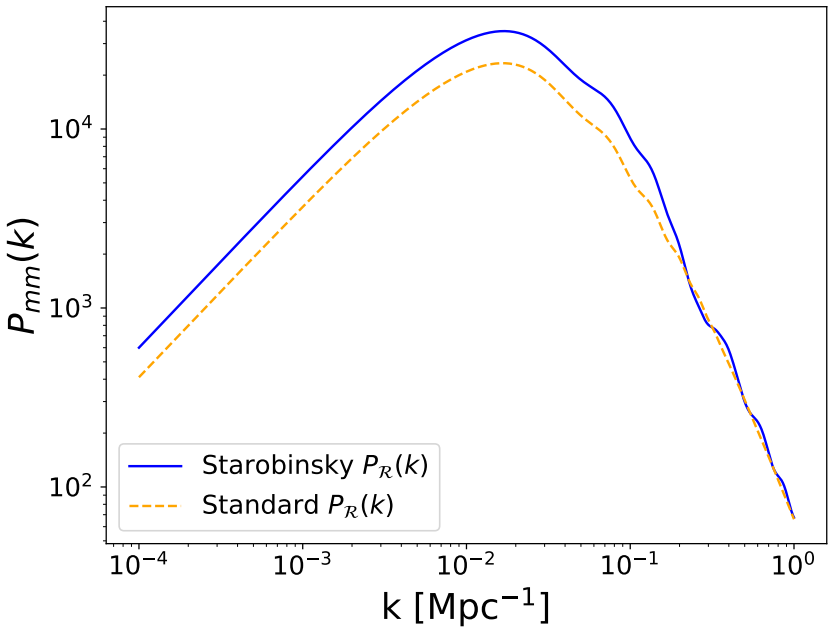


Figure 3:

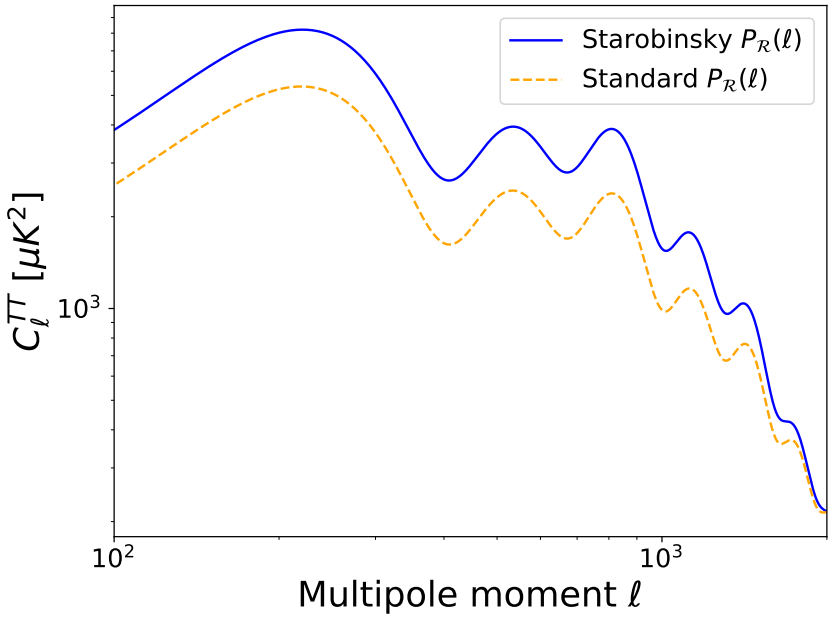


Figure 4:

To highlight that a difference in primordial power spectra can cause a change in both amplitude of late-time power as well as its 'shape', we plot the ratios.

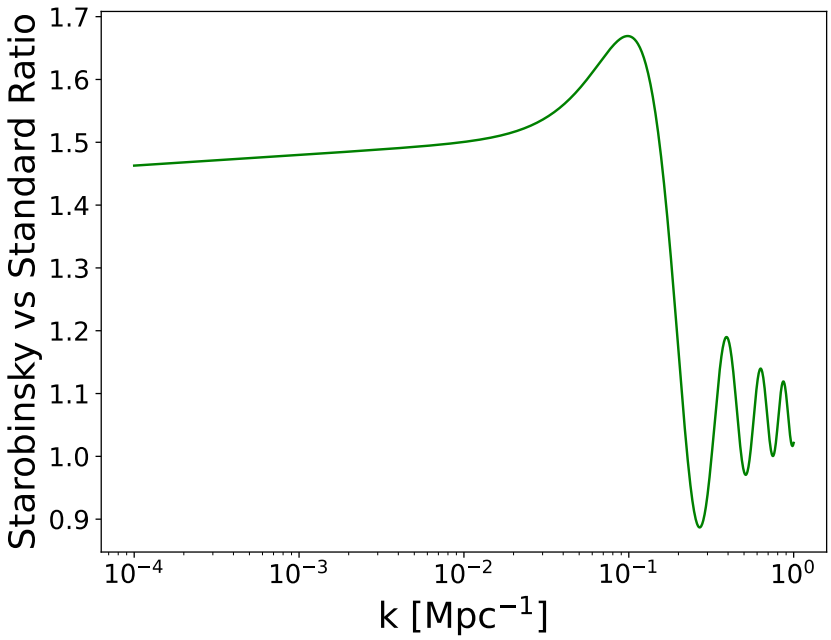


Figure 5: Ratio of the $P_{mm}(k)$ predicted by the Starobinsky spectrum compared to the standard power law formulation. Interestingly, this seems to produce another 'Starobinsky'-like shape, but I have not had time to understand why this is, or if it means anything.

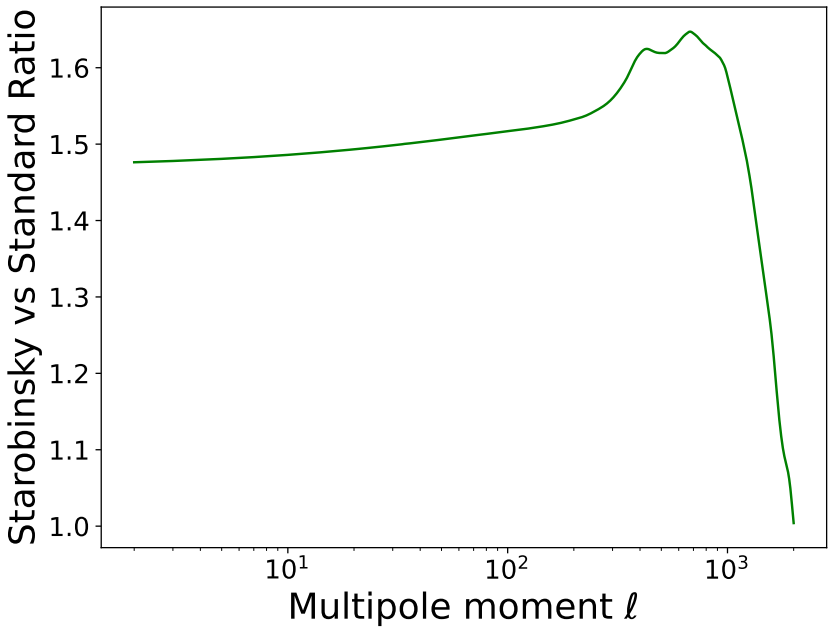


Figure 6: Ratio of the C_ℓ^{TT} predicted by the Starobinsky spectrum compared to the standard power law formulation.

2.3 Applications to the Cosmological Tensions

The paper by Terasawa et al. [2] conjoins the use of primordial power spectra modifications with Bayesian inference. Letting all the cosmological parameters, as well as those specifically used to formulate the spectrum, to be minimized over, they note that it results in insufficient power to alleviate the S_8 cosmological tension. This is given to be down to the fact that *Planck* data

has sufficient statistical power such that it can constrain the shape of the primordial spectrum to scales $k \sim 0.2\ \mathrm{Mpc}^{-1}$. This does not leave enough freedom in spectrum modifications to generate effects large enough to resolve the tension.

2.4 Brief Aside

I am currently beginning to scope out a research collaboration with a fellow student to bring together the physics of inflation with the field(s) of quantum chaos/complexity/information[6][7]. One direction I could imagine going in is seeking modifications to the PPS, so I thought it would also help to go over it here. Such a project is in its very early stages, and I know rather little about where it might lead. We are hoping to write a proposal/plan soon.

3 The Big Question; Research Avenues

Since the Physics department look for[8] some element of novel research in the marking of these dissertations, I feel it is important to continually keep an eye out for where this might come from. The first 5 weeks of this project (in term-time) have given be a good background in the area, and I believe this has been early enough to pursue more modern frontiers. Finding this gap, however, will be challenging in the time-frame, and in a way that doesn't send me down a 'rabbit-hole' where I end up spending 10 weeks learning QFT!

On this point, I propose Siri and I aim to have decided a rough direction for this novelty by our week 6 meeting. If agreed upon, this will result in the next week of my dissertation being a literature review to try to find interesting areas of research, and to scope out how viable they will be. My background in MCMC does mean that I can quickly relate any theoretical work back to late-time observations (thus constraining the theory), and so I believe I have the ability to explore areas that will require rather a lot of reading before any computation, if necessary. Of course, I will listen to my supervisor's advice where ever possible.

[6] This is what he is doing *his* dissertation on

[7] This in itself is in no way novel, but we are of course looking for novel *angles*.

[8] according to discussions with Prof. Matthew Turner, Prof. Gareth Alexander, and Prof. Jon Duffy,



Figure 7: **Happy Bonfire Night!!!**

References

[1] A. A. Starobinskij. "Spectrum of adiabatic perturbations in the universe when there are singularities in the inflationary potential." In: *Soviet Journal of Experimental and Theoretical Physics Letters* 55 (May 1, 1992). ADS Bibcode: 1992JETPL..55..489S, pp. 489–494. ISSN: 0021-3640. URL: https://ui.adsabs.harvard.edu/abs/1992JETPL..55..489S (visited on 11/04/2026).

[2] Ryo Terasawa et al. *Testing small-scale modifications in the primordial power spectrum with Subaru HSC cosmic shear, primary CMB and CMB lensing*. Mar. 26, 2025. DOI: 10.48550/arXiv.2503.20396. arXiv: 2503.20396[astro-ph.CO]. URL: http://arxiv.org/abs/2503.20396 (visited on 11/04/2026).